B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS

A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

MID-APRIL TEST 2026-27

MARKING KEY

APPLIED MATHEMATICS (241)

Class: XII **Time: 1hr**

Date:18/04/26 **Max Marks: 25**

Admission no: **Roll no:**

General Instructions:

Question 1 to 7 carries ONE mark each. Questions 8 to 10 carries TWO marks each. Questions 11 to 14 carries THREE marks each.

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1. The derivative of x^{2x} w.r.t.x is
   a) x^{2x-1} b) $2x^{2x}\log x$ **c) $2x^{2x}(1+\log x)$** d) $2x^{2x}(1-\log x)$
2. If $x= at^2$, $y= 2at$ then dy/dx is :
   a) **1/t** b) t c) $-1/t^2$ d) t^2
3. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to:
   a) $\pi unit$ **b) $1/\pi$ units** c) $\pi/2$units d) 2πunits
4. If the radius of a circle is increasing at the rate of 1.4cm/sec , then the rate of increasing in its circumference is:
   a) 2.8cm/sec b) 4.4cm/sec **c) 8.8cm/sec** d) 1.4cm/sec
5. Radius of a sphere is increasing at the rate of 2cm/sec. The rate of change of its volume , when its radius is 6cm is:
   a) **$288\pi cm^3/sec$** b) $8\pi cm^3/sec$ c) $12\pi cm^3/sec$ d) None of these

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

6. **Assertion**: The function $f(x) = e^{2x}$ is strictly increasing on R.
   Reason: f'(x) = > 0 for all real values of x. **a**
7. **Assertion**: The function $f(x) = (x+2)\,e^{-x}$ is increasing in the interval $(-1,\infty)$.
   Reason: A function f(x) is increasing, if $f'(x) > 0$. **d**

8. Find dy/dx, if $(x^2+y^2)^2 = xy$.

Sol:

$$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = x\frac{dy}{dx}+y$$

$$\frac{dy}{dx}\left[4y(x^2+y^2)-x\right] = y-4x(x^2+y^2)$$

$$\frac{dy}{dx} = \frac{y-4x(x^2+y^2)}{4y(x^2+y^2)-x}$$

$$\frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x}$$

9. If $e^y(x+y) = 1$, Show that dy/dx = $-e^y$.

Sol: $e^y + (x+1)e^y\frac{dy}{dx} = 0$ substitute $e^y(x+1) = 1$

$e^y + \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -e^y$

10. The radius of an air bubble is increasing at the rate of ½ cm per sec. At what rate is the volume of the bubble increasing when the radius is 1cm?

Sol:

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Substitute $r = 1$ cm and $\frac{dr}{dt} = \frac{1}{2}$ cm/s into the derived formula:

$$\frac{dV}{dt} = 4\pi(1)^2 \cdot \left(\frac{1}{2}\right)$$

$$\frac{dV}{dt} = 4\pi \cdot 1 \cdot \frac{1}{2}$$

$$\frac{dV}{dt} = 2\pi \text{ cm}^3/\text{s}$$

11. Find dy/dx , if $x^y = y^x$.

Sol:

Apply the natural log (ln) to both sides to simplify the exponents:

$$\ln(x^y) = \ln(y^x)$$

1. **Left side:** $\ln x \cdot \frac{dy}{dx} + y \cdot \frac{1}{x}$

2. **Right side:** $1 \cdot \ln y + x \cdot \frac{1}{y} \cdot \frac{dy}{dx}$

$$(\ln x)\frac{dy}{dx} - \frac{x}{y}\frac{dy}{dx} = \ln y - \frac{y}{x}$$

$$\frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x}$$

Create common denominators within the parentheses:

$$\frac{dy}{dx}\left(\frac{y\ln x - x}{y}\right) = \frac{x\ln y - y}{x}$$

$$\frac{dy}{dx} = \frac{x\ln y - y}{x} \cdot \frac{y}{y\ln x - x}$$

$$\frac{dy}{dx} = \frac{y(x\ln y - y)}{x(y\ln x - x)}$$

12. Differentiate $\frac{\sqrt{x}\,(x+4)^{3/2}}{(4x-3)^{4/3}}$.

Sol:

$$\ln(y) = \ln\left(\frac{\sqrt{x}(x+4)^{3/2}}{(4x-3)^{4/3}}\right)$$

Using log properties $\ln(\frac{a \cdot b}{c}) = \ln(a) + \ln(b) - \ln(c)$:

$$\ln(y) = \frac{1}{2}\ln(x) + \frac{3}{2}\ln(x+4) - \frac{4}{3}\ln(4x-3)$$

Differentiate with respect to x using the chain rule ($\frac{d}{dx}[\ln(u)] = \frac{1}{u} \cdot \frac{du}{dx}$):

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + \frac{3}{2(x+4)} - \frac{4(4)}{3(4x-3)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + \frac{3}{2(x+4)} - \frac{16}{3(4x-3)}$$

Combine the fractions on the right side:

$$\frac{1}{y}\frac{dy}{dx} = \frac{3(x+4)(4x-3) + 9x(4x-3) - 32x(x+4)}{6x(x+4)(4x-3)}$$

$$\frac{dy}{dx} = \frac{\sqrt{x}(x+4)^{3/2}}{(4x-3)^{4/3}} \cdot \frac{2(4x^2 - 29x - 9)}{3x(x+4)(4x-3)}$$

13. Differentiate: x^x w.r.t xlogx.

Sol:

Differentiate $u = x^x$ with respect to x:

- Take the natural logarithm of both sides: $\ln u = \ln(x^x) = x\ln x$.
- Differentiate both sides with respect to x:

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x\ln x)$$

$$\frac{du}{dx} = x^x(1 + \ln x)$$

Differentiate $v = x\ln x$ with respect to x:

- Apply the product rule:

$$\frac{dv}{dx} = \frac{d}{dx}(x\ln x) = x \cdot \frac{d}{dx}(\ln x) + \ln x \cdot \frac{d}{dx}(x)$$

$$\frac{dv}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dv}{dx} = 1 + \ln x$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{x^x(1 + \ln x)}{1 + \ln x}$$

$$\frac{du}{dv} = x^x$$

14. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground , away from the wall at the rate of 2cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

Sol:

$$x^2 + y^2 = L^2$$

$$x^2 + y^2 = 5^2 = 25$$

When the foot of the ladder is 4 m away ($x = 4$):

$$4^2 + y^2 = 25$$

$$16 + y^2 = 25 \implies y^2 = 9 \implies y = 3\,\text{m}$$

To find the rates of change, we differentiate both sides of the Pythagorean equation:

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(25)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Divide the equation by 2 and plug in the known values:

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$(4)(0.02) + (3)\frac{dy}{dt} = 0$$

$$0.08 + 3\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{0.08}{3} \approx -0.0267\,\text{m/sec}$$

Converting this back to cm/sec:

$$\frac{dy}{dt} = -\frac{8}{3}\,\text{cm/sec} \approx -2.67\,\text{cm/sec}$$